Risk-Adjusted Project Costs Estimation in the Black-Scholes Framework

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Abstract

The purpose of this article is to propose a new method for risk-adjusted estimation of project module (task) costs. The main concept underlying this method is to utilize Nobel-prize winning framework developed by Myron Scholes and Fischer Black in 1973 to value European call and put options on stocks. This approach attempts to answer a question, "How much money should we quote for a project that consists of unstable modules (tasks) when each module has its own unique volatility?" The method assumes that developing and committing to cost estimates for a project (especially a fixed-price project) is very similar to being in a short position in European call option where only a scenario when the actual cost is larger than the estimated cost has a significant impact on the project status. The methodology proposed is quite simple and requires a spreadsheet application or, even, a simple scientific calculator to be operational.

Keywords

Cost Estimation, Budgeting, Risk Management, Cost Variability.

Introduction

Overview of Project Cost Estimation Methodologies

According to the Guide to the Project Management Body of Knowledge (Project Management Institute, Inc., 2004) there are several cost estimating techniques available to project managers in their budgeting efforts. These methods include:

- Analogous or top-down estimating,
- Parametric modeling,
- Bottom-up estimating,
- Computerized tools (e.g. Monte Carlo simulation), and
- Other cost estimating methods such as vendor bid analysis.

Analogous estimating uses actual costs of previous, similar projects tasks in order to forecast current project task costs. Parametric modeling, on the other hand, uses previous projects' metrics (usually per unit) to determine new project costs. Bottom-up estimating requires project manager to estimate the cost of each project module (or task) individually and then roll up those numbers to derive the total project cost estimate. Monte Carlo simulation allows project managers to imitate various scenarios of a real-life project, especially when other analyses are too complex mathematically or too difficult to reproduce (Kim Heldman, 2002).

The common factor of most of these techniques is that they attempt to estimate future project costs based on the scale of similar tasks performed before and usually **ignore task volatilities**. Some methods suggest adding 10-20% contingency premium to each estimated module or task cost (Kim Heldman, 2002). However, these numbers are usually based on practical experience rather than scientifically justified methods. As a result, these methods are rather imprecise and could lead to serious under- or over-estimation of the project budget. For example, Jørgensen and Sjøberg (2004) report that according to the Standish Group (see www.standishgroup.com) survey, average software project cost overrun was as high as 189% of the original estimate, and only 17% of the projects were completed on-time, on budget and with all the features and functions as initially specified.

Another weakness of this approach is, **what** is being considered a "similar task". It is obvious that even tasks with identical titles taken from two different projects will not necessarily have the same size, duration and cost. For example, "painting the home" tasks can vary greatly with respect to cost whether it is a two-bedroom apartment or a large mansion.

Financial industry has long used various methodologies to incorporate stock price and variability in asset valuation models. One of the most famous methods was proposed by Merton Black and John Scholes in 1973 in their article "The Pricing of Options and Corporate Liabilities," published in the "Journal of Political Economy" (Black and Scholes, 1973)

Background on Options and Black-Scholes Model

Options on stocks were first traded on organized exchange in 1973. Since then, due to a dramatic growth in option markets, options are now traded at many exchanges throughout the world. The underlying assets include stocks, indices, foreign currencies, debt instruments, commodities and future contracts (Hull, 1997 and Dubofsky, 1992)

There are two basic types of options – a **call** option and a **put** option. For the purposes of this paper we will focus reader's attention on the call options (see Charts 1 and 2). A call option gives the holder a right to buy the underlying asset by a certain date at a certain price. The price in the contract is usually referred to as **exercise price** or **strike price** and the date is usually called **expiration date** or **maturity**. European options can only be exercised on the maturity date. The person is said to be in the "long position" if he purchased the option and in "short position" if he sold an option.

If the person has taken a long position in a European call, his/her payoff can be expressed as:

$$Payoff = \max(S_T - X, 0) \text{ (Chart 1)} \tag{1}$$

If the person has taken a short position in a European call, his/her payoff can be expressed as:

$$Payoff = \min(X - S_T, 0) \text{ (Chart 2)} \tag{2}$$

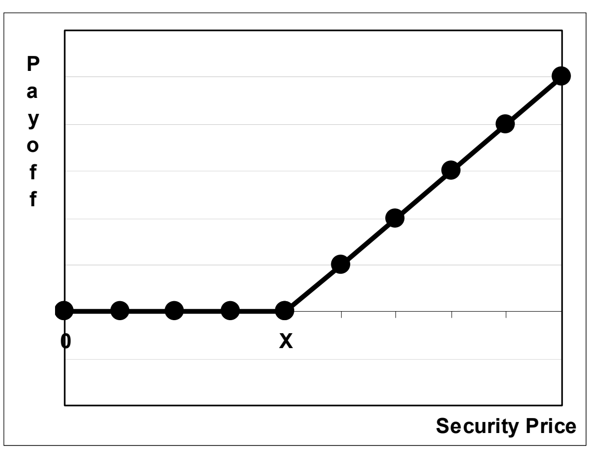

Chart 1: European Call Option (Long)

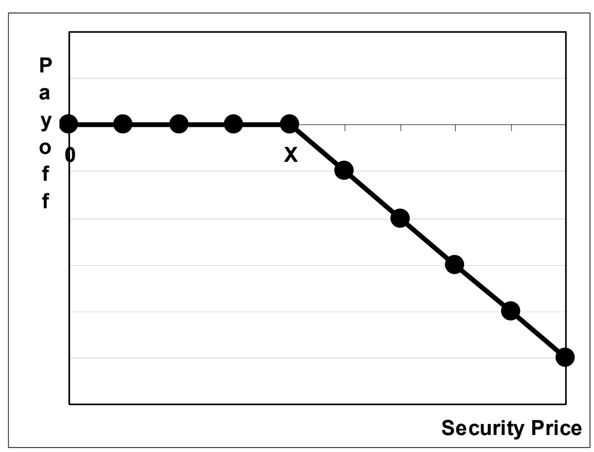

Chart 2: European Call Option (Short)

In their Nobel prize-winning paper Black and Scholes (Black and Scholes, 1973) proposed a model to value the European call options. Their calculation resulted in the following set of equations:

$$c = SN(d_1) - Xe^{-rT}N(d_2),$$

(3)

and

$$d_1 = \frac{\ln\left(S_0/X\right) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

(4)

$$d_2 = d_1 - \sigma\sqrt{T}$$

(5)

where

c – European call option value

S_0 – underlying asset price at time $t = 0$

X – exercise (strike) price of the call option

r – risk-free interest rate

σ – standard deviation of stock returns

T – time to expiration

$N(d)$ – cumulative standard normal distribution function

Black-Scholes model proved to be very versatile and was later modified to value American options (can be exercised at any point of time before maturity) as well as options on dividend-paying stocks, indices, currencies and futures.

Methodology

The model will be introduced in the following manner: first the assumptions underlying the model will be stated and explained, then the set of formulas will be derived, and, finally, a numerical example will be presented.

Assumptions

1. Module costs and implied work efforts cannot be split and/or have some parts of them removed once the valuation model has been used,
2. Project module duration is fixed,
3. Module cost estimates' relative changes (i.e. "returns") are normally distributed:

$$\frac{dEPMC}{EPMC} \sim N\left(0, \sigma\sqrt{\Delta t}\right),$$

4. Module cost estimates follow a lognormal distribution (follows from Assumption 3),
5. Module costs estimates follow a continuous Itô process (see Chart 3):

$$\frac{dEPMC}{EPMC} = \mu dt + \sigma dz\ , \tag{6}$$

6. While the historical cost estimates of "similar" modules are not particularly relevant, the degree of their estimated variations is (e.g. similar tasks tend to have different costs but comparable variability of relative changes),
7. Project managers are penalized (i.e. pay) for being over the budget but are indifferent if the actual project module cost is lower than the estimated project module cost. In other words, committing to project cost estimates (this is especially true for fixed-price projects) is very **similar in nature to being in a short position in the European call option** where only an increase in cost has an impact on the project budget and status:

$$\textit{Payoff (Loss)} = \textit{min}\ [EPMC - APMC, 0] \tag{7}$$

where

EPMC – Estimated project module cost

APMC – Actual project module cost.

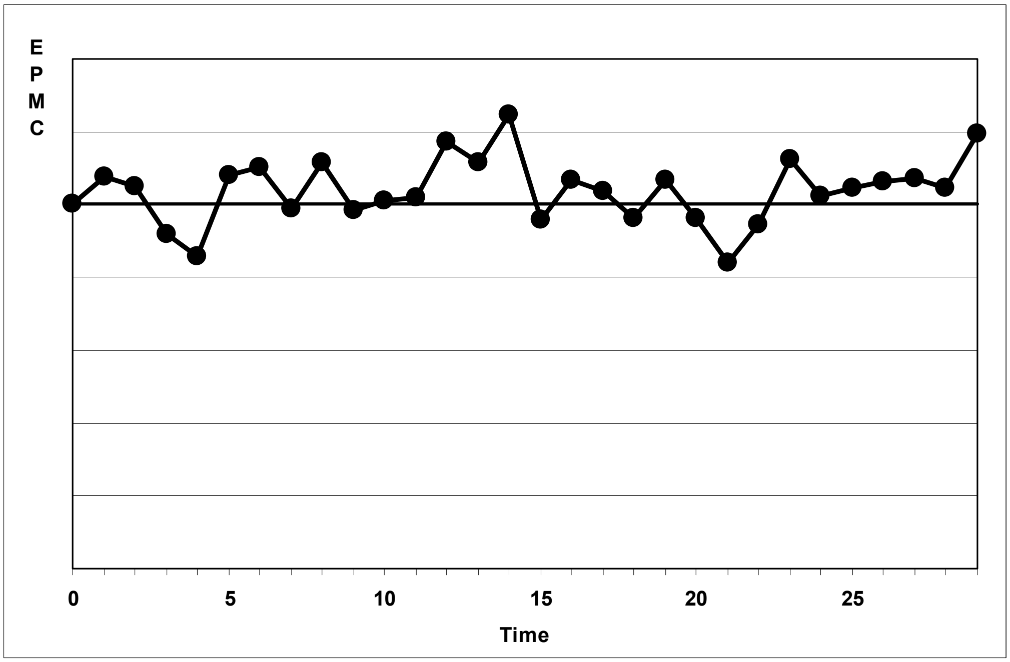

Chart 3: Estimated Project Module Cost Variability

While assumptions 1-5 are relatively straightforward, the last two assumptions require further explanation and justification.

Explanation for Assumption 6:

Intuitively all project managers know that some tasks, like, for example, project charter creation, are usually relatively stable and do not exhibit high degrees of variation from project to project. These tasks may vary in their absolute monetary values depending on the project size and complexity but the variances of their relative changes (i.e. “returns” in financial context) are usually approximately equal. On the other hand, expenses associated with quality assurance stages like testing or debugging can vary greatly due to unpredictability and volatility of these phases. It is therefore, logical to assume that similar tasks may have the same degrees of relative variability even if their absolute sizes and absolute standard deviations may be quite different.

Explanation for Assumption 7:

Firstly, empirical studies demonstrate that lower than expected actual costs rarely occur. For example, Connolly and Dean (1997) report that the actual effort used by student programmers to perform certain programming tasks fell inside their 98% confidence effort prediction intervals in only about 60% of the cases. Moreover, even after specific training, the proportion inside the minimum and maximum boundaries increased only to about 70%.

Furthermore, Jørgensen, Teigen and Molokken (2004) conducted a study of software development task estimates of 195 student projects and 49 industry projects. Students were able to correctly include only 62% of actual efforts, while the industry experts estimated only 35% of actual effort values.

Jørgensen and Teigen (2002) conducted an experiment in which 12 software experts were required to provide 90% confidence effort estimation intervals on 30 previously completed tasks. These professionals were asked to supply their estimates in sets of 10 and after each set of estimates was submitted, they were provided with actual efforts these tasks required. Although “90% confident”, the professionals included, on average, only 64% of the actual effort values on the first 10 tasks, 70% on the next 10 tasks and, 81% on the last group of tasks. This implies that even after 20 tasks with feedback, there was a systematic bias towards overconfidence.

Jørgensen (2004c) studied cost forecasts provided by seven estimation teams. All projects tasks the professionals were required to assess belonged to the organization they worked at. However, only 43% of the team effort estimates included the actual effort.

Furthermore, other studies dealing with effort estimates in non-IT industries were conducted over the years. Majority of them including Tversky and Kahneman (1974), Alpert and Raiffa (1982), Kahnemann, Slovic et al. (1982) and Yaniv and Foster (1997) confirm that there is a strong bias towards overconfidence in task effort estimation. Lichtenstein and Fischhoff (1977) report that levels of overconfidence seem to have no relationship with skill or experience levels. Therefore they argue that levels of overconfidence should not be expected to diminish with greater experience.

Also, Jørgensen and Sjørberg (2004) argue that even if the initial estimate was too high, the remaining effort is usually spent trying to improve the delivered product (Parkinson’s Law).

It should be noted that the model could have a slight overestimation bias with respect to the risk premium value. However, this situation is preferable to the opposite scenario. It is also preferable to adding 10-20% contingency funds on the "ad-hoc" basis to every project task.

Model Description

In this model (as in any regular project) all the work is subdivided into smaller tasks or modules and the duration of each individual module is being estimated. Once the duration of each project module (henceforth, *PM*) is determined, estimated project module costs (henceforth, *EPMC*) can be forecasted. The sum of all project module costs will represent the total project cost:

$$ETPC = \sum_{j=1}^{k} EPMC_j \tag{8}$$

where

ETPC – estimated total project cost

EPMC – estimated project module cost

k – number of project modules

However, as was mentioned before, estimated project module costs (*EPMC*s) are not reliable and subject to constant changes and adjustments. We, therefore suggest adjusting the *EPMC*s for risk associated with their volatility by calculating a risk premium value (henceforth *RPV*).

$$EPMC' = EPMC + RPV \tag{9}$$

where

EMPC'- estimated project module cost (risk-adjusted)

EPMC - estimated project module cost

RPV – risk premium value

By employing the Black-Scholes model and treating this situation as a short position in European-style call we get:

$$RPV = EPMC * N(d_1) - TPMC * e^{-WACC*T} N(d_2)$$

(10)

$$d_1 = \frac{\ln\left(EPMC / TPMC\right) + \left(WACC + \sigma^2 / 2\right)T}{\sigma\sqrt{T}}$$

(11)

$$d_2 = d_1 - \sigma\sqrt{T}$$

(12)

where:

RPV - risk premium value

EPMC – estimated project module cost

TPMC – threshold project module cost

WACC – weighted average cost of capital

σ – standard deviation of relative cost changes

T – time remaining to the project module completion

In order to make this model more suitable to a project management reality we can simplify the model be assuming that:

EPMC = TPMC

WACC = 0 (since the time value of money is traditionally ignored in project cost estimation)

By making these changes we transform the formulas to:

$$RPV = EPMC(N(d_1) - N(d_2))$$

(13)

$$d_1 = \frac{\sigma\sqrt{T}}{2}$$

(14)

$$d_2 = d_1 - \sigma\sqrt{T} = -\frac{\sigma\sqrt{T}}{2}$$

(15)

Numerical Scenario

Assume that we have a project that can be subdivided into two project modules – PM_1 and PM_2. Once each module's duration has been estimated we can start estimating the project module costs by taking the following steps:

1. Estimate each project module cost by using traditional PMBOK techniques (e.g. bottom-up estimating)
2. Estimate the annualized standard deviation of historical relative changes of similar project modules
3. Calculate the *RPV* of each module by using formulas (13), (14) and (15)
4. Calculate the risk-adjusted estimated project module cost (*EMPC'*) for each module
5. Calculate the total risk-adjusted project cost

$$ETPC' = \sum_{j=1}^{k} EPMC'_j$$

(16)

Let's assume that we have conducted all the preliminary analysis and that:

$EPMC_1$ = \$100,000

$EPMC_2$ = \$75,000

T_1 = 3 months

T_2 = 2 months

We now have to examine the variability of similar tasks from past projects. In this example we rely on historical data from several project modules from previous projects to assess the cost estimate variability of these types of project modules.

PM_1 in this example has two corresponding (or similar) modules from past projects: Project 1 and Project 2 (see Table 1). We assume that these modules' initial *EPMC*s were \$100,000 and \$200,000 respectively. As time progressed, the cost estimates were adjusted and recorded at times t = 30, 60 and 90. Based on the on the revised estimates, relative changes in estimated costs for that particular period are calculated in the following manner:

$$y_n = \ln\left(EPMC_t \big/ EPMC_{t-n} \right) \tag{17}$$

Period (e.g. 30-day in this example) relative changes in cost estimates are later converted to annual rates:

$$y_{annual} = e^{y_n \frac{360}{n}} - 1 \tag{18}$$

In the final stage we calculate total annualized standard deviation based on relative changes of both modules. The standard deviation is the measure of cost estimate volatility of this particular type of module (i.e. PM_1 in this case). Note, that since we use “relative” volatility measure in this method, the standard deviation can be calculated based on observations from several historical data sets. This would have been impossible if we used absolute or monetary standard deviation.

The standard deviation for PM_1 equals 163.12%. Similarly we can calculate the historical annual standard deviation for PM_2 which equals 51.61% (see Tables 1 and 2). It is easy to see from the data provided in Tables 1 and 2 that PM_2 is a less volatile process than PM_1; thus its variance is much lower than that of PM_1.

	Time	Historical Module Costs (estimated at 30-day intervals)	Percentage Changes (per period) $y_n = \ln\left(EPMC_t / EPMC_{t-n} \right)$	Percentage Changes (annual, implied) $y_{annual} = e^{y_n \frac{360}{n}} - 1$
PROJECT 1				
	t = 0	$100,000.00		
	t = 30	$110,000.00	9.53%	213.84%
	t = 60	$125,000.00	12.78%	363.67%
	t = 90	$130,000.00	3.92%	60.10%
PROJECT 2				
	t = 0	$200,000.00		
	t = 30	$190,000.00	-5.13%	-45.96%
	t = 60	$213,000.00	11.43%	294.01%
	t = 90	$225,000.00	5.48%	93.03%
	t = 120	$237,000.00	5.20%	86.55%
			Annual σ_{Total}	**163.12%**

Table 1: Standard Deviation Estimation for Project Module 1 - *PM*$_1$

	Time	Historical Module Costs (estimated at 30-day intervals)	Percentage Changes (per period) $y_n = \ln\left(EPMC_t / EPMC_{t-n} \right)$	Percentage Changes (annual, implied) $y_{annual} = e^{y_n \frac{360}{n}} - 1$
PROJECT 3				
	t = 0	$50,000.00		
	t = 30	$52,000.00	3.92%	60.10%
	t = 60	$54,000.00	3.77%	57.28%
	t = 90	$56,000.00	3.64%	54.71%
	t = 120	$61,234.00	8.94%	192.18%
	t = 150	$62,000.00	1.24%	16.09%
	t = 180	$63,000.00	1.60%	21.17%
	t = 210	$64,000.00	1.57%	20.80%
	t = 240	$65,000.00	1.55%	20.45%
	t = 270	$66,000.00	1.53%	20.11%
	t = 300	$67,000.00	1.50%	19.78%
	t = 330	$68,000.00	1.48%	19.46%
	t = 360	$68,000.00	0.00%	0.00%
			Annual σ_{Total}	**51.61%**

Table 2: Standard Deviation Estimation for Project Module 1 – *PM*$_2$

By substituting these results into formulas (13), (14) and (15) we can calculate the *RPV*s for both modules (see Table 3).

	Project Module 1	Project Module 2
EPMC	$ 100,000.00	$ 75,000.00
σ_{Annual}	163.12%	51.61%
T	0.2500	0.1667
d_1	0.4078	0.1053
d_2	-0.4078	-0.1053
$N(d_1)$	0.6583	0.5419
$N(d_2)$	0.3417	0.4581
RPV	$ 31,657.92	$ 6,292.13
EPMC'	$ 131,657.92	$ 81,292.13

Table 3: Risk Premium Calculation

And finally, by using formula (9) we can derive the risk-adjusted project module costs for PM_1 and PM_2:

$EPMC'_1$ = $131.657.92

$EPMC'_2$ = $81,292.13

Therefore, the estimated total project cost (risk adjusted) will be equal to $212,950.05.

Conclusion

The purpose of this paper was to derive a methodology for the risk-adjusted project cost estimation by employing the Black-Scholes framework for valuing European call options. The two key assumptions underlying this methodology are:

1. Similar tasks have similar volatilities (i.e. variance of relative changes), and
2. Committing to the project cost estimate (baseline) is very similar to being in a short position on a European call option.

As a result, a set of formulas was derived to assist project managers in estimation of the risk premium value for each project model cost estimate. Furthermore, the theoretical analysis was supplemented by a sample case study, that clearly demonstrates the effectiveness and simplicity of the model proposed.

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